Hydraulic systems modelling

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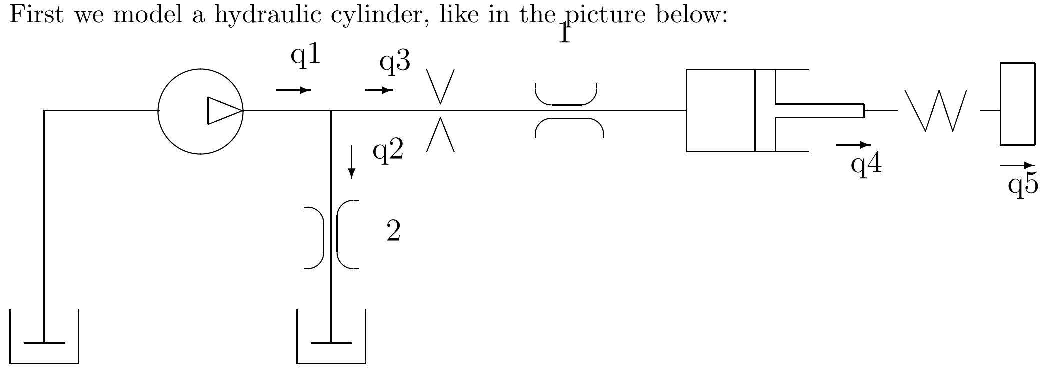
First we model a hydraulic cylinder, like in the picture below:



We assume an ideal flow source , so that $f_1 = f_1^s(t)$, which will become our first kinematic constraint

$$\Psi_1 := f_1 - f_1^s(t) = 0$$

and there are no other sources in this stage. The flow f_1 should be equal to the flows $f_2 + f_3$. Since the cylinder is a reservoir to the fluid we want this to become a constraint on displacement variables. This is done by integrating the flow constraints and setting the initial values for q_1, q_2 and q_3. Initial values for q_1 and q_2 are insignificant and we can set them to zero. q_{30} is the initial value for q_3, and the constraint becomes

$$\Phi_1 := q_1 - q_2 - q_3 + q_{30} = 0$$

There is a turbular flow orifice like shown in the figure. In ideal case this follows the following law

$$f_3 = C_d A \sqrt{\frac{2}{\rho}(p_1 - p_2)}$$

where p_1 is the flow pressure before orifice and p_2 is the flow pressure after the orifice. We can associate an effort to this pressure difference $e_3 = p_1 - p_2$ and using this we have

$$f_3 = C_d A \sqrt{\frac{2}{\rho} e_3} \Leftrightarrow$$
$$e_3 = \frac{\rho}{2C_d^2 A^2} f_3^2$$

if we idealize this resistor, so that it resists in both flow directions, we get

$$e_3 = \frac{\rho}{2C_d^2 A^2} |f_3| f_3$$

and this contributes one term to content function. Remembering the definition of content we get

$$D = \int edf = \int \frac{\rho}{2C_d^2 A^2} |f_3| f_3 df_3 = \frac{\rho}{2C_d^2 A^2} |f_3|^3$$

Then we move to those two laminar orifices that are marked with two opposite arcs in the figure. For ideal laminar orifice

$$f_3 = C_{lam_1}(p_2 - p_3) = C_{lam_1} e_4 \Leftrightarrow$$
$$e_4 = \frac{f_3}{C_{lam_1}}$$

where e_4 is again an effort associated with the pressure difference around the laminar orifice 1. Using again the definition of the content function we get

$$D = \int edf = \int \frac{f_3}{C_{lam_1}} df_3 = \frac{1}{2C_{lam_1}} f_3^2$$

Similarly treating the laminar orifice 2 we get

$$f_2 = C_{lam_2}(p1 - 0) = C_{lam_2} e_2 \Leftrightarrow$$
$$D = \frac{1}{2C_{lam_2}} f_2^2$$

For the spring connected into the cylinder, we have ideally

$$V = \frac{1}{2} k (q_4 - q_5)^2$$

where k is the spring constant and q_4 the displacement of the spring and is zero when the cylinder piston is at rest at time $t = 0$. Variable q_5 is the displacement of the wall and since this is not moving, we have yet another constraint

$$\Phi_2 := q_5 = 0$$

Even though this variable q_5 seems rather ridiculous it is taken into account in order to get future development more straightforward. If the fluid was incompressible the model for cylinder would be

$$q_3 = q_4 A_c + V_0$$

and this is simply interpreted as 'The volume of fluid flowed into the cylinder is equal to the initial volume of the cylinder plus the changed volume of the cylinder'. Using compressible fluids, the model for the cylinder becomes

$$f_3 = \frac{dV}{dt} + \frac{V}{\beta} \frac{dp_3}{dt}$$

where V is the volume of the cylinder and β is a constant. Volume V is the initial volume summed with the changed volume, that is

$$V = q_4 A_c + V_0$$

also the pressure p_3 gives the force $p_3 A_c$ which pushes the cylinder to the right and the spring pushes the cylinder to the left with kq_4 and so

$$p_3 A_c = kq_4 \Leftrightarrow$$
$$p_3 = \frac{k}{A_c} q_4$$

Summing these results up we have

$$f_3 = \frac{d}{dt}(q_4 A_c + V_0) + \frac{q_4 A_c + V_0}{\beta} \frac{d}{dt}(\frac{k}{A_c} q_4) = \left(A_c + \frac{k(q_4 A_c + V_0)}{\beta A_C} \right) \frac{dq_4}{dt} = \left(A_c + \frac{k(q_4 A_c + V_0)}{\beta A_C} \right) f_4$$

and thus our second constraint becomes

$$\Psi_2 := \left(A_c + \frac{k(q_4 A_c + V_0)}{\beta A_C} \right) f_4 - f_3 = 0$$

Now after we list all the constants we have all done.

$$T^* = 0$$
$$V = \frac{1}{2} k (q_4 - q_5)^2$$
$$D = \frac{\rho}{2 C_d^2 A^2} |f_3|^3 + \frac{1}{2 C_{lam_1}} f_3^2 + \frac{1}{2 C_{lam_2}} f_2^2$$
$$\Phi_1 := q_1 - q_2 - q_3 + q_{30} = 0$$
$$\Phi_2 := q_5 = 0$$
$$\Psi_1 := f_1 - f_1^s(t) = 0$$
$$\Psi_2 := \left(A_c + \frac{k(q_4 A_c + V_0)}{\beta A_c} \right) f_4 - f_3 = 0$$

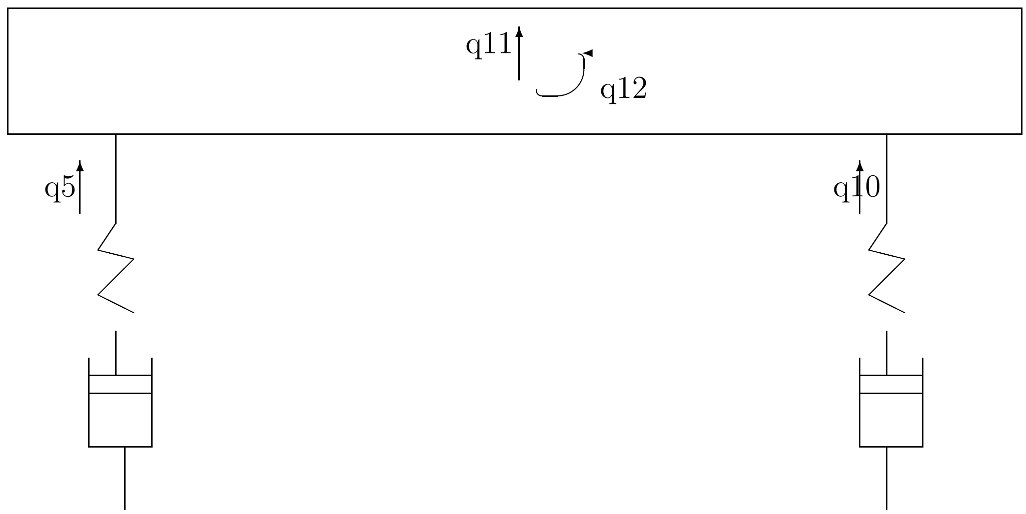
$$C_d = 0.61$$
$$C_{lam_1} = 2 \cdot 10^{-7}$$
$$C_{lam_2} = 3 \cdot 10^{-8}$$
$$\beta = 700 \cdot 10^6$$
$$A_c = 0.01$$
$$k = 500000$$
$$V_0 = 0.025$$
$$\rho = 1000$$
$$q_{30} = V0$$
$$q_{40} = 0$$
$$A = 0.03t$$

The code for Dynast is listed here: `* SYSTEM;`

```
fS1/pulse/L1=0.125, L2=0, TD=0.04, TR=0, TT=0.01, TF=0; :Flow source
Cd = 0.61; :Discharge coefficient
Clam1 = 2*10**(-7); :Laminar flow gain
```

```
Clam2 = 3*10**(-8); :Laminar flow gain
beta = 700*10**6; :(Pa)
Ac = 0.01; :Cylinder area
k = 500000; :Spring constant
V0 = 0.025; :Initial volume
rho = 1000; :Fluid density
q30 = V0; :Initial volume
A/poly/0,0.03; :Area of the flow orifice

:  Equations of motion
SYSVAR q1,q2,q3,q4,q5,f1,f2,f3,f4,f5,mu1,mu2,kappa1,kappa2;
0 = VD.q1 - f1;
0 = VD.q2 - f2;
0 = VD.q3 - f3;
0 = VD.q4 - f4;
0 = VD.q5 - f5;

0 = kappa1 + mu1;
0 = 1/Clam2*f2 - kappa1;
0 = rho/(2*Cd*A(TIME)**2)*f3*abs(f3)+1/Clam1*f3 + mu2 - kappa1;
0 = k*(q4-q5) -( Ac + k*(q4*Ac+V0)/(beta*Ac) )*mu2;
0 = -k*(q4-q5) + kappa2;

:  Algebraic equations
0 = f1-fS1(TIME);
0 = f3-( Ac + k*(q4*Ac+V0)/(beta*Ac) )*f4;
0 = q1-q2-q3+q30;
0 = q5;

:  Simulation
*TR;
tr 0.00001 0.1; :  time=0..0,1 s
INIT q1=0, q2=0, q3=0.025, q4=0;
PRINT(100) q1,q2,q3,q4,f2,f3;
RUN;

*END;
```

The final model should be two similar hydraulic systems as modelled above, but instead of the wall we had a moving block, where both of the hydraulic systems were connected. The figure below illustrates this system



Nice feature of this modelling system is that we don't have to rethink the models for the hydraulic systems, only the kinematics of the block. We assign two new variables to the block, namely q_{11}, which is the displacement of the block's centre of mass in x-direction and q_{12}, which is the angular orientation of the block. We assume that q_{12} get small values and using this we have

$$q_{12} = \frac{q_{10} - q_5}{L}$$

which transforms into a new constraint

$$\Phi_3 := q_{10} - q_5 - Lq_{12} = 0$$

where L is the length of the block. Also, since q_5 and q_{10} are displacements of the connecting points in x-direction we have

$$q_{11} = \frac{q_5 + q_1 0}{2} \Leftrightarrow$$
$$\Phi_4 := q_5 + q_{10} - 2q_{11} = 0$$

The block contributes to kinetic energy both with translational and rotational movement, these terms are

$$T^* = \frac{1}{2}M\dot{q}_{11}^2 + \frac{1}{2}J\dot{q}_{12}^2$$

where M is the mass of the block and J is the block inertia. We also have a force acting on the block, namely the gravitational force Mg. This has to be taken into account when calculating the equations of motion. We have a new system of DAEs as

$$T^* = \frac{1}{2}M\dot{q}_{11}^2 + \frac{1}{2}J\dot{q}_{12}^2$$

$$V = \frac{1}{2}k(q_4 - q_5)^2$$

$$D = \frac{\rho}{2C_d^2A^2}|f_3|^3 + \frac{1}{2C_{lam_1}}f_3^2 + \frac{1}{2C_{lam_2}}f_2^2$$

$$\Phi_1 := q_1 - q_2 - q_3 + q_{30} = 0$$

$$\Phi_2 := q_5 = 0$$

$$\Phi_3 := q_{10} - q_5 - Lq_{12} = 0$$

$$\Phi_4 := q_5 + q_{10} - 2q_{11} = 0$$

$$\Psi_1 := f_1 - f_1^s(t) = 0$$

$$\Psi_2 := \left(A_c + \frac{k(q_4A_c + V_0)}{\beta A_c}\right) f_4 - f_3 = 0$$

$$C_d = 0.61$$

$$C_{lam_1} = 2 \cdot 10^{-7}$$

$$C_{lam_2} = 3 \cdot 10^{-8}$$

$$\beta = 700 \cdot 10^6$$

$$A_c = 0.01$$

$$k = 500000$$

$$V_0 = 0.025$$

$$\rho = 1000$$

$$q_{30} = V0$$

$$q_{40} = 0$$

$$A = 0.03t$$

$$M = 2500$$

$$L = 5$$

$$J = 100$$

$$g = 9.81$$

This system of DAEs are written in DYNAST form as follows

```
   * SYSTEM;
   fS1/pulse/L1=0.125, L2=0, TD=0.04, TR=0, TT=0.01, TF=0; :  Flow
source
fS6/pulse/L1=0.125, L2=0.125, TD=0.04, TR=0, TT=0.01, TF=0; :  Flow
source
Cd = 0.61; :Discharge coefficient
Clam1 = 2*10**(-7); :Laminar flow gain
Clam2 = 3*10**(-8); :Laminar flow gain
beta = 700*10**6; :(Pa)
Ac = 0.01; :Cylinder area
k = 500000; :Spring constant
```

```
V0 = 0.025; :Initial Volume
rho = 1000; :Fluid density
M = 2500; :Mass of the block
Izz = 100; :Block inertia
L = 5; :Length of the block
g = 9.81; :guess
q30 = V0; :Initial volume for the cylinder 1
q80 = V0; :Initial volume for the cylinder 2
A1/poly/0,0.03; :Orifice area 1
A2/poly/0,0.03; :Orifice area 2

:  Equations of motion for cylinder 1
SYSVAR q1,q2,q3,q4,q5,q6,q7,q8,q9,q10,q11,q12,f1,f2,f3,f4,f5,f6,f7,f8,f9,f10,f11,f12,mu1,mu2,
0 = VD.q1 - f1;
0 = VD.q2 - f2;
0 = VD.q3 - f3;
0 = VD.q4 - f4;
0 = VD.q5 - f5;

0 = kappa1 + mu1;
0 = 1/Clam2*f2 - kappa1;
0 = rho/(2*Cd*A1(TIME)**2)*f3*abs(f3)+1/Clam1*f3 + mu2 - kappa1;
0 = k*(q4-q5) -( Ac + k*(q4*Ac+V0)/(beta*Ac) )*mu2;
0 = -k*(q4-q5) - kappa11 + kappa12;

:  Algebraic equations
0 = f1-fS1(TIME);
0 = f3-( Ac + k*(q4*Ac+V0)/(beta*Ac) )*f4;
0 = q1-q2-q3+q30;


:  Equations of motion for cylinder 2
0 = VD.q6 - f6;
0 = VD.q7 - f7;
0 = VD.q8 - f8;
0 = VD.q9 - f9;
0 = VD.q10 - f10;

0 = kappa6 + mu6;
0 = 1/Clam2*f7 - kappa6;
0 = rho/(2*Cd*A2(TIME)**2)*f8*abs(f8)+1/Clam1*f8 + mu7 - kappa6;
0 = k*(q9-q10) -( Ac + k*(q9*Ac+V0)/(beta*Ac) )*mu7;
0 = -k*(q9-q10) + kappa11 + kappa12;

:  Algebraic equations
0 = f6-fS6(TIME);
0 = f8-( Ac + k*(q9*Ac+V0)/(beta*Ac) )*f9;
0 = q6-q7-q8+q80;

:  Block equations
```

```
0 = VD.q11 - f11;
0 = VD.q12 - f12;

0 = M*VD.f11 - 2*kappa12 - M*g;
0 = Izz*VD.f12 - L*kappa11;
0 = q10 - q5 - L*q12;
0 = q5 + q10 - 2*q11;


:  simulation
*TR;
tr 0.00001 0.1; :  time=0..1 s
INIT q3=0.025, q8=0.025;
PRINT(100) f2,f3,f7,f8,q1,q5,q10,q11,q12,f11,f12;
RUN;

*END;
```