73154 Topics in applied mathematics
Modelling and Computer Simulation of Multidomain Physical Systems

HYDRAULIC BOOM

Simulation exercise report 14.12.1999

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1. Introduction

Course's absolute demand for the DAE modelling exercise was that it should belong to at least two different physical domains (mechanics, fluid mechanics, electric circuits, etc.). Therefore a hydromechanical boom application was selected as our exercise work. Also an another wish was that this exercise should be a real research problem, but in our case this hydraulic boom is just a toy case and there is no real lift boom to compare with.

The picture of this imaginary boom mechanism is presented in the figure 1a. The system consist two booms and two hydraulic cylinders. The booms are assumed as rigid parts and the hydraulic cylinders do not have mass properties. The end position is seen in the figure 1b and the task is to simulate the lift sequence. Both hydraulic cylinders belong to the same hydraulic circuit and when the valve is opened the booms are moving simultaneously. The simulation also includes some kind of friction modelling for the slider pieces, which are in real booms for example nylon material.

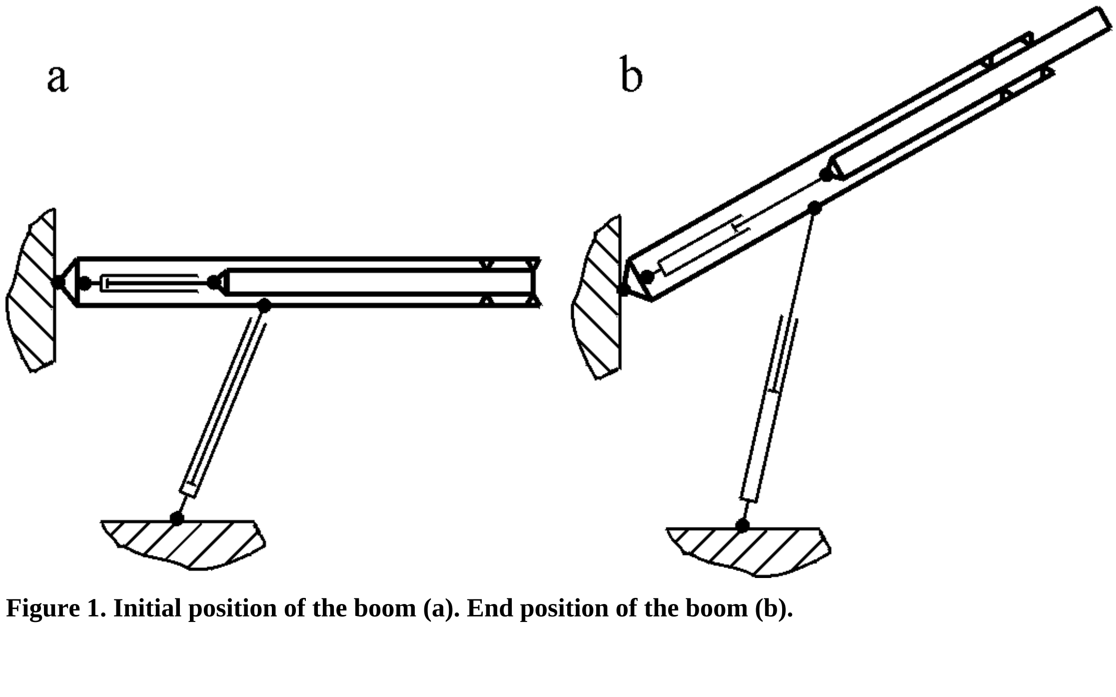

Figure 1. Initial position of the boom (a). End position of the boom (b).

Two different simulation models were constructed: a classical ODE model and a DAE model. The ODE model was simulated by using a MATLAB® m-file (by using the ode45-function) and The DAE model by using the DYNAST-program via internet.

2. ODE model of the boom

2.1 Nomenclature

The basic variables are presented in the figure 2. The global origin is at the point A.

g	constant of gravity
m_1	mass of boom 1
m_2	mass of boom 2
x	piston movement of cylinder 1
θ	rotation angle of boom 1 (and boom 2)
L_1	length of boom 1
L_2	length of boom 2
I_{1A}	moment of inertia of boom 1 about point A
I_2	moment of inertia of boom 2 about center of gravity
I	total moment of inertia
N_1, N_2	normal forces of slider pieces
N	total normal force
d	distance between slider pieces
μ	friction coefficient of slider pieces
A_1	piston are of cylinder 1
A_2	piston are of cylinder 2
p_1	pressure of cylinder 1
p_2	pressure of cylinder 2
F_1	cylinder force 1
F_2	cylinder force 2
r	position of fastening point of cylinder 2 along boom 1
$\mathbf{r}$	position vector of fastening point of cylinder 2 on boom 1
$\mathbf{r}_0$	initial position vector of fastening point of cylinder 2 on boom 1
$\mathbf{R}$	rotation matrix
x_p	x-coordinate of fastening point of cylinder 2 on ground
y_p	y-coordinate of fastening point of cylinder 2 on ground
$\mathbf{e}_2$	unit direction vector of cylinder 2
$\mathbf{M}_{F2}$	moment vector of cylinder force 2
M_{F2}	magnitude of $\mathbf{M}_{F2}$
x_4	piston movement of cylinder 2
L_4	length of cylinder 2
L_{40}	initial length of cylinder 2
y_i	first order ODE transformation variables
$\mathbf{z}$	first order ODE variable vector
B	bulk modulus of oil
p_s	supply line pressure
q_{nom}	nominal volumetric flow
Δp_{nom}	nominal pressure difference
$f(t)$	control voltage function (dimensionless)
V_0	initial volume (pipes and hoses)

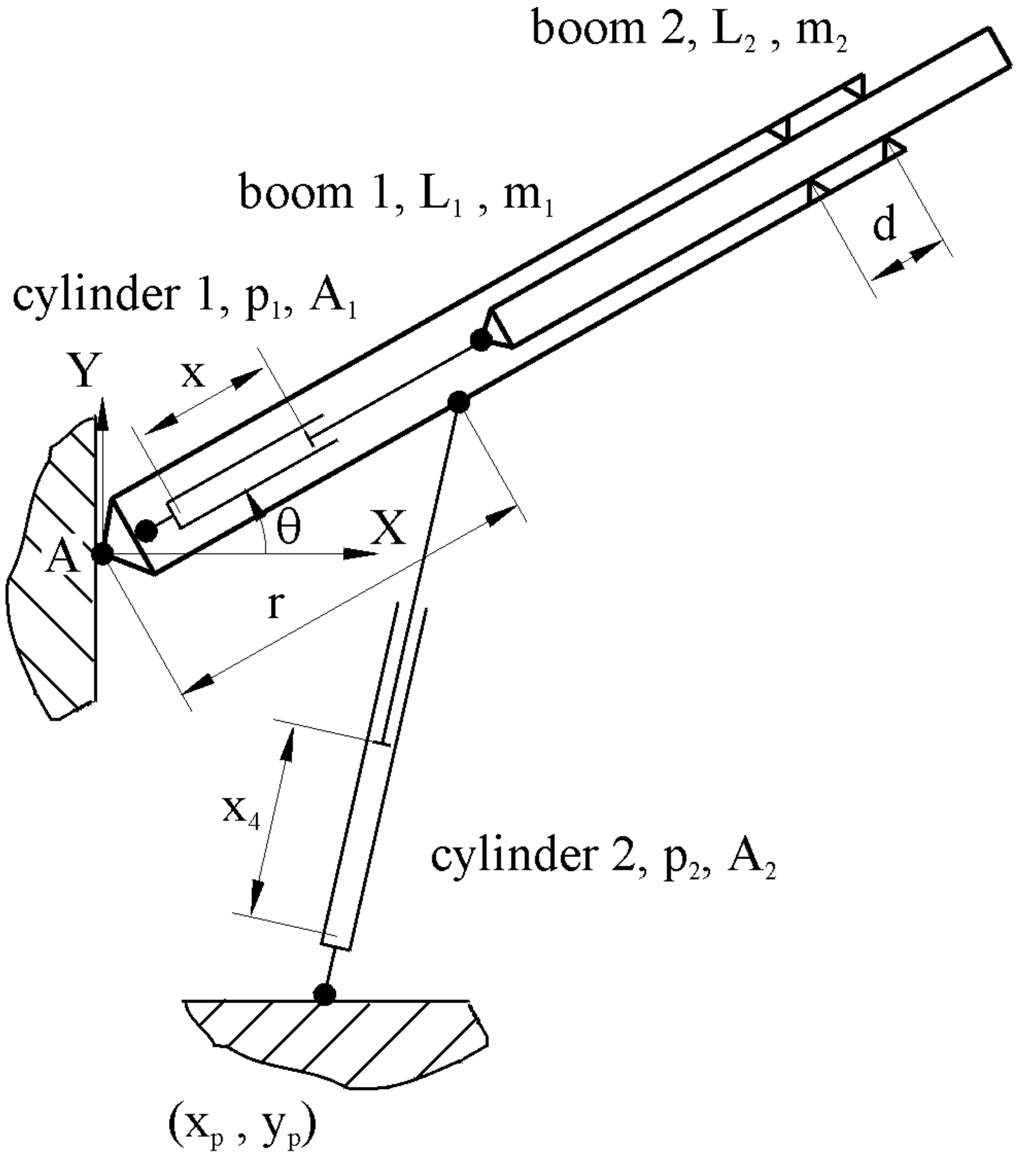

Figure 2. The basic variables of the boom.

2.2 Derivation of equations

The derivation was made by using Newtonian mechanics. The free body picture of the boom 2 is presented in figure 3:

The normal forces can be solved from equations

$$\begin{gathered} N_1 + N_2 - m_2 g\cos\theta - m_2(L_1 - \frac{L_2}{2} + x)\ddot{\theta} = 0 \\ N_1(L_2 - x - d) + N_2(L_2 - x) - m_2 g\cos\theta\,\frac{L_2}{2} - m_2(L_1 - \frac{L_2}{2} + x)\frac{L_2}{2}\ddot{\theta} - I_2\ddot{\theta} = 0 \end{gathered} \tag{1}$$

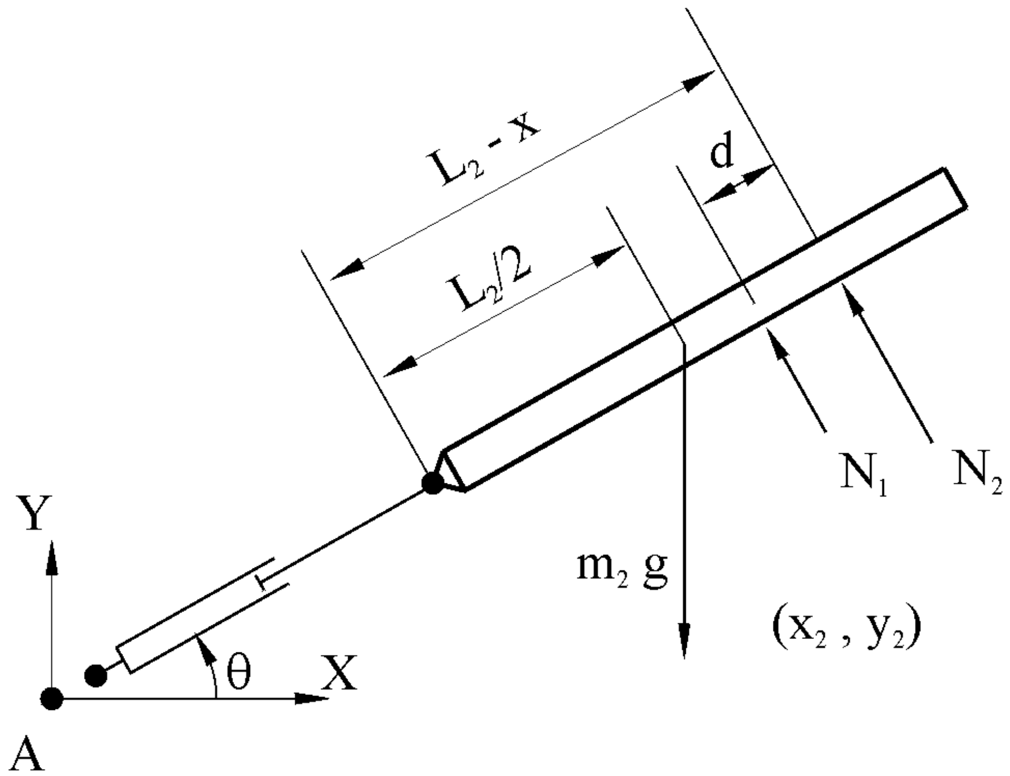

Figure 3. The free body picture of the boom 2.

and these give

$$N_1 = \{m_2 g \cos\theta + m_2 (L_1 - \frac{L_2}{2} + x)\ddot{\theta}\} \frac{(\frac{L_2}{2} - x)}{d} - \frac{I_2 \ddot{\theta}}{d} \qquad (2)$$
$$N_2 = m_2 g \cos\theta + m_2 (L_1 - \frac{L_2}{2} + x)\ddot{\theta} - N_1$$

The total normal force for the slider pieces friction is

$$N = |N_1| + |N_2| \qquad (3)$$

and the equation of motion of the boom 2 is then simply

$$m_2 \ddot{x} = F_1 - \mu N - mg \sin\theta \qquad (4)$$

and

$$F_1 = p_1 A_1 \qquad (5)$$

The friction model used assumes that the direction of the friction force does not change ie. that the boom 2 will move only forward. This assumption is not true at the very beginning of the simulation when hydraulic forces are not yet enough large to overcome gravitational forces. However, too complex friction model was not reasonable due numerical difficulties and this simple model was used. An another possibility to overcome this direction problem is to firstly determine a static balance position for the system (meaning that there is some pressure in cylinders) and then start the simulation. Nevertheless, in our simulation we let the boom fall from the initial position until hydraulic forces take the control.

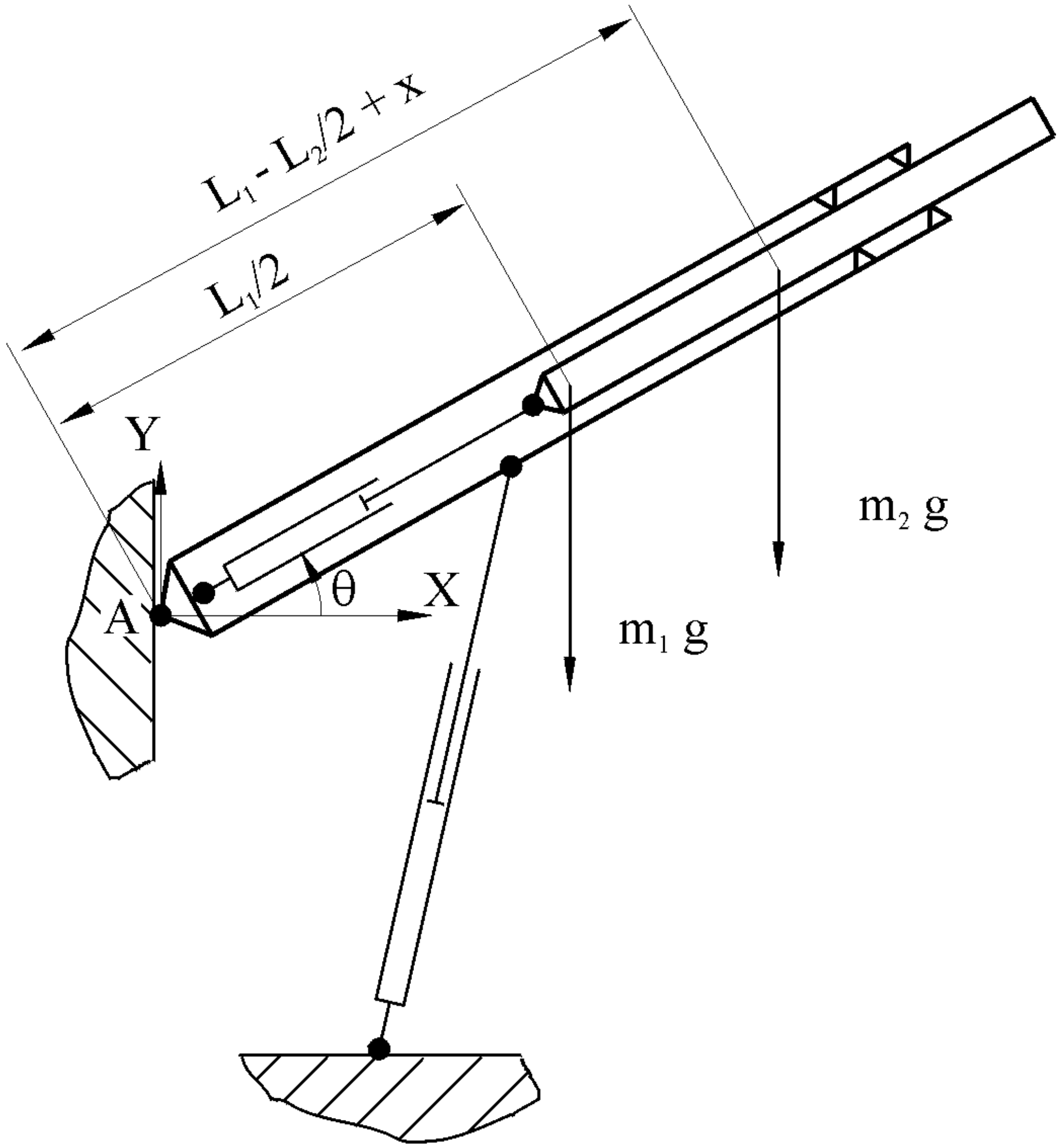

Figure 4. The free body picture of the boom 1.

The free body picture of the boom 1 in presented in figure 4: The position of the fastening point of the hydraulic cylinder 2 on the boom 1 is

$$\mathbf{r} = \mathbf{R}\mathbf{r}_0 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} r \\ 0 \end{bmatrix} = r \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \tag{6}$$

and using this the unit direction vector of the cylinder 2 is

$$\mathbf{e}_2 = \frac{\mathbf{r} - \mathbf{x}_p}{\left\| \mathbf{r} - \mathbf{x}_p \right\|} \tag{7}$$

The moment of the hydraulic force of the cylinder 2 can be calculated by the cross product

$$\mathbf{M}_{F_2} = \mathbf{r} \times F_2 \mathbf{e}_2 = M_{F_2} \mathbf{k} \tag{8}$$

where

$$F_2 = p_2 A_2 \tag{9}$$

By simplifying the equation (8) the magnitude of the moment is expressed as

$$M_{F_2} = \frac{r(x_p \sin\theta - y_p \cos\theta)}{L_4} F_2 \tag{10}$$

where

$$L_4 = \left\| \mathbf{r} - \mathbf{x}_p \right\| = \sqrt{(r\cos\theta - x_p)^2 + (r\sin\theta - y_p)^2} \tag{11}$$

The non-linear moment of inertia of the whole boom system is

$$I = I_{1A} + I_2 + m_2 (L_1 - \frac{L_2}{2} + x)^2 \tag{12}$$

and the angular equation of the motion is

$$I\ddot{\theta} = M_{F_2} - m_1 g \cos\theta \frac{L_1}{2} - m_2 g \cos\theta (L_1 - \frac{L_2}{2} + x) \tag{13}$$

The piston movement of the hydraulic cylinder 2 is

$$x_4 = \left\| \mathbf{r} - \mathbf{x}_p \right\| - L_{40} = L_4 - L_{40} \tag{14}$$

where

$$L_{40} = \left\| \mathbf{r}_0 - \mathbf{x}_p \right\| = \sqrt{(r - x_p)^2 + y_p{}^2} \tag{15}$$

The time derivative of (14) is

$$\dot{x}_4 = \frac{r(x_p \sin\theta - y_p \cos\theta)\dot{\theta}}{L_4} \tag{16}$$

The pressure of an hydraulic cylinder is determined by the equation

$$\dot{p} = \frac{B(\sqrt{\frac{p_s - p}{\Delta p_{nom}}} q_{nom} f(t) + \dot{V})}{V + V_0} \tag{17}$$

where V is the volume of the cylinder. The equation (17) is a simplified version of the equations presented in [1] and it is only for one-sided hydraulic movement (one hydraulic push). The dimensionless function *f(t)* represents the control voltage of the hydraulic valve. The function *f(t)* used in the simulation is presented in the figure 5.

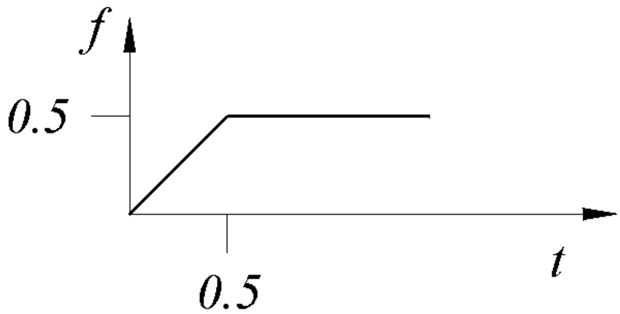

Figure 5. The control function *f* of the hydraulic valve.

Finally, we can collect the system of ODEs

$$
\begin{aligned}
m_2\dot{y}_1 &= p_1A_1 - \mu N - mg\sin\theta \\
I\dot{y}_2 &= M_{F_2} - m_1g\cos\theta\frac{L_1}{2} - m_2g\cos\theta(L_1 - \frac{L_2}{2} + x) \\
\dot{x} &= y_1 \\
\dot{\theta} &= y_2 \\
\dot{p}_1 &= \frac{B(\sqrt{\frac{p_s - p_1}{\Delta p_{nom}}}q_{nom}f(t) - A_1y_1)}{A_1x} \\
\dot{p}_2 &= \frac{B(\sqrt{\frac{p_s - p_2}{\Delta p_{nom}}}q_{nom}f(t) - A_2\dot{x}_4)}{A_2x_4}
\end{aligned}
\tag{18}
$$

and the ODE variable vector is

$$z = \begin{bmatrix} y_1 & y_2 & x & \theta & p_1 & p_2 \end{bmatrix}^T \tag{19}$$

2.3 Simulation results of ODE model

The MATLAB® m-files used in the simulation are presented in the appendix A. Some time domain plots are in the figure 6 and the picture of the lift sequence is in the figure 7.

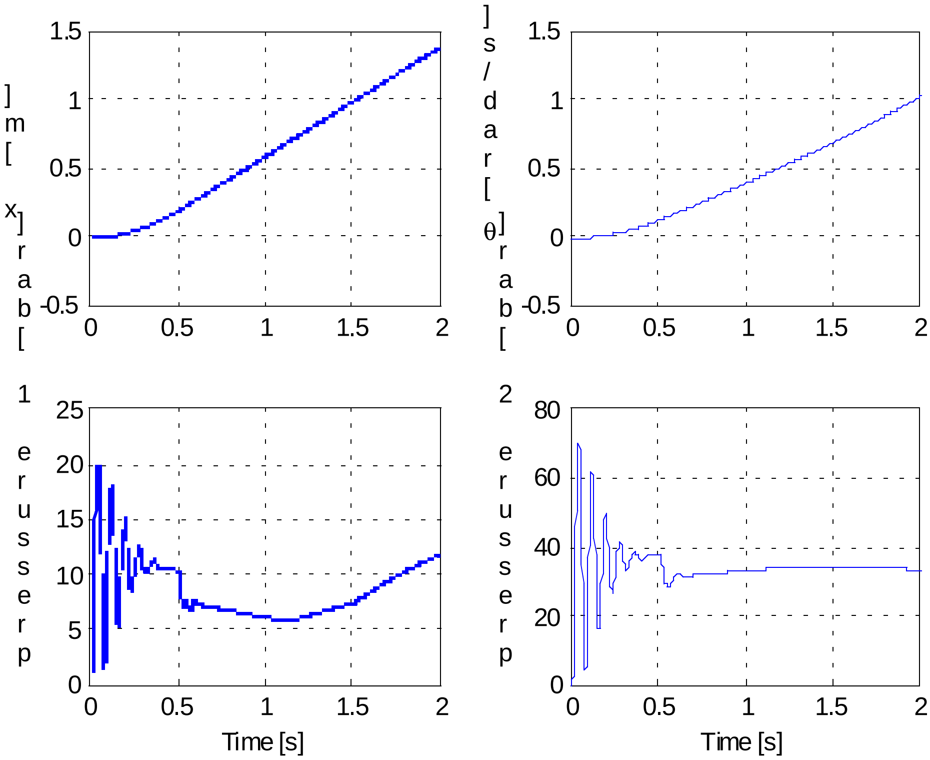

Figure 6. ODE simulation results.

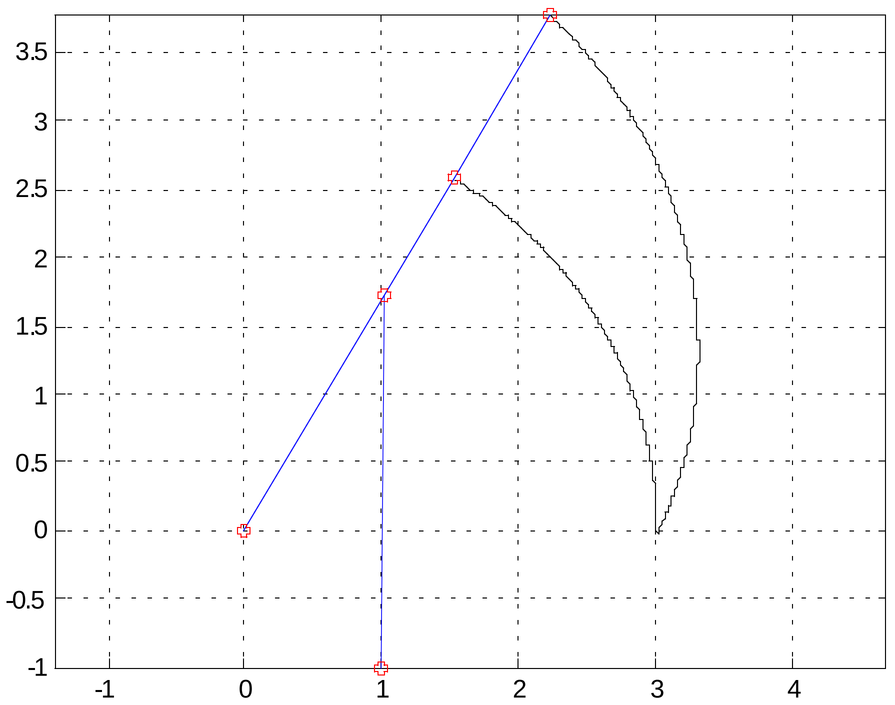

Figure 7. The lift sequence and the end position of the boom.

3. DAE model of the boom

3.1 Nomenclature

Most of the variables used here are same as in the chapter 2. The basic variables are presented in the figure 8.

q_1	x-coordinate of the center of boom 1
q_2	y-coordinate of the center of boom 1
q_3	angular coordinate of the boom 1 (θ)
q_4	x-coordinate of the center of boom 2
q_5	y-coordinate of the center of boom 2
q_6	angular coordinate of the boom 2
q_7	x-coordinate of the fastening point of cylinder 2 on boom 1
q_8	y-coordinate of the fastening point of cylinder 2 on boom 1
q_9	x-coordinate of the end of boom 1
q_{10}	y-coordinate of the end of boom 1
f_1	time derivative of q_1
f_2	time derivative of q_2
f_3	time derivative of q_3
f_4	time derivative of q_4
f_5	time derivative of q_5
f_6	time derivative of q_6
f_7	time derivative of q_7
f_8	time derivative of q_8
f_9	time derivative of q_9
f_{10}	time derivative of q_{10}
F_{1x}	x-component of cylinder force 1
F_{1y}	y-component of cylinder force 1
F_{2x}	x-component of cylinder force 2
F_{2y}	y-component of cylinder force 2
$F_{\mu x}$	x-component of friction force
$F_{\mu y}$	y-component of friction force
I_1	moment of inertia of boom 1 about center of gravity
I_2	moment of inertia of boom 2 about center of gravity

3.2 Derivation of equations

The kinetic coenergy of the system is

$$T^* = \frac{1}{2}m_1\dot{q}_1^{\,2} + \frac{1}{2}m_1\dot{q}_2^{\,2} + \frac{1}{2}I_1\dot{q}_3^{\,2} + \frac{1}{2}m_2\dot{q}_4^{\,2} + \frac{1}{2}m_2\dot{q}_5^{\,2} + \frac{1}{2}I_2\dot{q}_6^{\,2} \qquad (20)$$

and the potential energy is

$$V = m_1 g q_2 + m_2 g q_5 \qquad (21)$$

The kinematic constraints are (8 kappas)

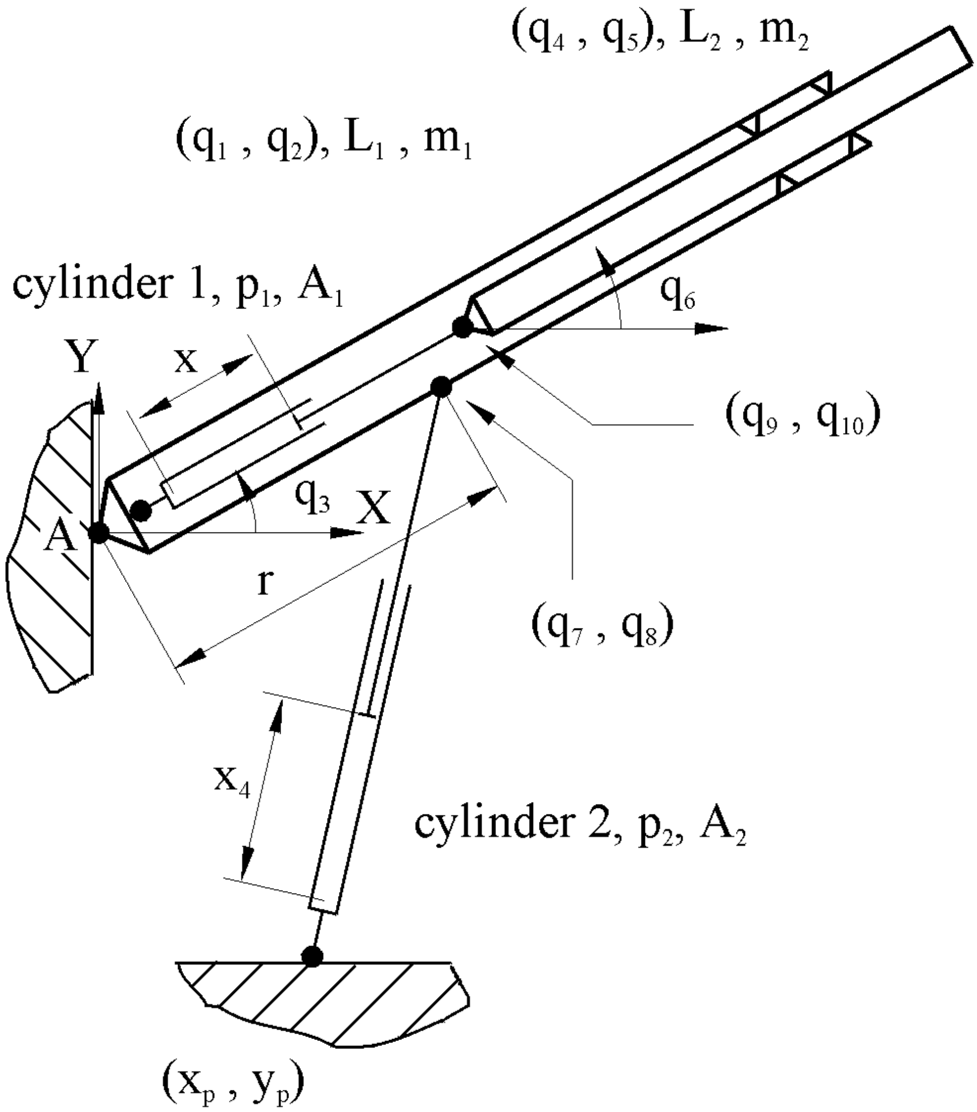

Figure 8. The basic variables of the boom.

$$
\begin{aligned}
\phi_1 &= q_1 - \frac{L_1}{2}\cos(q_3) = 0 \\
\phi_2 &= q_2 - \frac{L_1}{2}\sin(q_3) = 0 \\
\phi_3 &= q_5 - q_4 \tan(q_6) \\
\phi_4 &= q_3 - q_6 = 0 \\
\phi_5 &= q_7 - r\cos(q_3) = 0 \\
\phi_6 &= q_8 - r\sin(q_3) = 0 \\
\phi_7 &= q_9 - (q_4 - \frac{L_2}{2}\cos(q_6)) = 0 \\
\phi_8 &= q_{10} - (q_5 - \frac{L_2}{2}\sin(q_6)) = 0
\end{aligned}
\qquad (22)
$$

The constraint equation ϕ_3 is rather easy way to force the boom 2 into the slider movement wanted, but there might be problems with the tan-function if the angle q_6 closes to 90°. For our simulation purposes, however, this is a very well acceptable limitation.

The applied efforts are

$$\begin{aligned} Q_1 &= 0 \\ Q_2 &= 0 \\ Q_3 &= 0 \\ Q_4 &= 0 \\ Q_5 &= 0 \\ Q_6 &= 0 \\ Q_7 &= F_{2x} \\ Q_8 &= F_{2y} \\ Q_9 &= F_{1x} - F_{\mu x} \\ Q_{10} &= F_{1y} - F_{\mu y} \end{aligned} \tag{23}$$

Applying to the equation (3.28) from [2]

$$\begin{gathered} m_1 \dot{f}_1 + \kappa_1 = 0 \\ m_1 \dot{f}_2 + m_1 g + \kappa_2 = 0 \\ I_1 \dot{f}_3 + \kappa_1 \frac{L_1}{2} \sin(q_3) - \kappa_2 \frac{L_1}{2} \cos(q_3) + \kappa_4 + \kappa_5 r \sin(q_3) - \kappa_6 r \cos(q_3) = 0 \\ m_2 \dot{f}_4 - \kappa_3 \tan(q_6) - \kappa_7 = 0 \\ m_2 \dot{f}_5 + m_2 g + \kappa_3 - \kappa_8 = 0 \\ I_1 \dot{f}_6 - \kappa_3 q_4 (1 + \tan(q_6)^2) - \kappa_4 - \kappa_7 \frac{L_2}{2} \sin(q_6) + \kappa_8 \frac{L_2}{2} \cos(q_6) = 0 \\ \kappa_5 = F_{2x} \\ \kappa_6 = F_{2y} \\ \kappa_7 = F_{1x} - F_{\mu x} \\ \kappa_8 = F_{1y} - F_{\mu y} \end{gathered} \tag{24}$$

where

$$\begin{aligned} \dot{q}_1 - f_1 &= 0 \\ \dot{q}_2 - f_2 &= 0 \\ \dot{q}_3 - f_3 &= 0 \\ \dot{q}_4 - f_4 &= 0 \\ \dot{q}_5 - f_5 &= 0 \\ \dot{q}_6 - f_6 &= 0 \\ \dot{q}_7 - f_7 &= 0 \\ \dot{q}_8 - f_8 &= 0 \\ \dot{q}_9 - f_9 &= 0 \\ \dot{q}_{10} - f_{10} &= 0 \end{aligned} \tag{25}$$

The effort constraints are

$$
\begin{aligned}
F_{1x} &= \cos(q_6)\, p_1 A_1 \\
F_{1y} &= \sin(q_6)\, p_1 A_1 \\
F_{2x} &= \frac{q_7 - x_p}{L_4}\, p_2 A_2 \\
F_{2y} &= \frac{q_8 - y_p}{L_4}\, p_2 A_2 \\
F_{\mu x} &= \cos(q_6)\mu N \\
F_{\mu y} &= \sin(q_6)\mu N
\end{aligned} \tag{26}
$$

and the dynamic constrains

$$
\begin{aligned}
\dot{p}_1 &= \frac{B(\sqrt{\frac{p_s - p_1}{\Delta p_{nom}}} q_{nom}\, f(t) - A_1 \dot{x}}{A_1 x + V_0} \\
\dot{p}_2 &= \frac{B(\sqrt{\frac{p_s - p_2}{\Delta p_{nom}}} q_{nom}\, f(t) - A_2 \dot{x}_4)}{A_2 x_4 + V_0}
\end{aligned} \tag{27}
$$

The auxiliary equations are

$$
N = |N_1| + |N_2| \tag{28}
$$

$$
\begin{aligned}
N_1 &= \{m_2 g \cos(q_6) + m_2 (L_1 - \frac{L_2}{2} + x)\, \dot{f}_6\} \frac{(\frac{L_2}{2} - x)}{d} - \frac{I_2 \dot{f}_6}{d} \\
N_2 &= m_2 g \cos(q_6) + m_2 (L_1 - \frac{L_2}{2} + x)\, \dot{f}_6 - N_1
\end{aligned} \tag{29}
$$

$$
x = \sqrt{q_9{}^2 + q_{10}{}^2} - (L_1 - L_2) \tag{30}
$$

$$
\dot{x} = \frac{q_9 f_9 + q_{10} f_{10}}{\sqrt{q_9{}^2 + q_{10}{}^2}} \tag{31}
$$

$$
L_4 = \sqrt{(q_7 - x_p)^2 + (q_8 - y_p)^2} \tag{32}
$$

$$
x_4 = L_4 - L_{40} \tag{33}
$$

$$
\dot{x}_4 = \frac{(q_7 - x_p)\, f_7 + (q_8 - y_p)\, f_8}{L_4} \tag{34}
$$

3.3 Simulation results of DAE model

The DYNAST input file of the simulation is presented in the appendix B. The simulation aborted at time 0.73 seconds instead of desire 2 seconds because of the analysis time limitation of 250 seconds in the DYNAST program. The DAE results are compared with the ODE results in the figures 9 and 10 and some minor differences can be seen in the hydraulic pressure curves. The reason for this remains unknown, because DAE and ODE input files should be ok.

4. Conclusions

The ODE solution produced very compact and rather elegant set of equations (18). The basic difficulty with the ODEs was the derivation procedure, because one have to actually use his/her brains to get statements correct. The solution with MATLAB® was quite straightforward.

The DAE model derivation was extremely fun to make. It produced a huge number of equations (22, 24, 25, 26 and 27) but none of them was too difficult to obtain. One especially pleasant feature was that the moment statements were not required in the equations. The only problem was the equations of the normal forces of boom 2 (29), because no easy way to calculate them was found. The solution with the DYNAST was rather easy and only real difficulties were misprints in the input file and the user interface.

Finally, the analysis times were 2 seconds with the MATLAB® in Pentium III 400 MHz and 250 second with the DYNAST via internet (at the simulation abort time of 0.73 seconds). These analysis times may not be comparable, because we do not know details about CPU-time distribution of the DYNAST machine, but if this ratio of 1:680 holds, it is a real DAE killer.

Anyway, our boom got simulated.

References

[1] Erno Keskinen, Jori Montonen, Sirpa Launis and Michel Cotsaftis, *Simulation of Wire and Chain Mechanisms in Hydraulic Driven Booms*, IASTED 1999, Australia

[2] Richard A. Layton, *Principles of Analytical System Dynamics*, Springer, 1998.

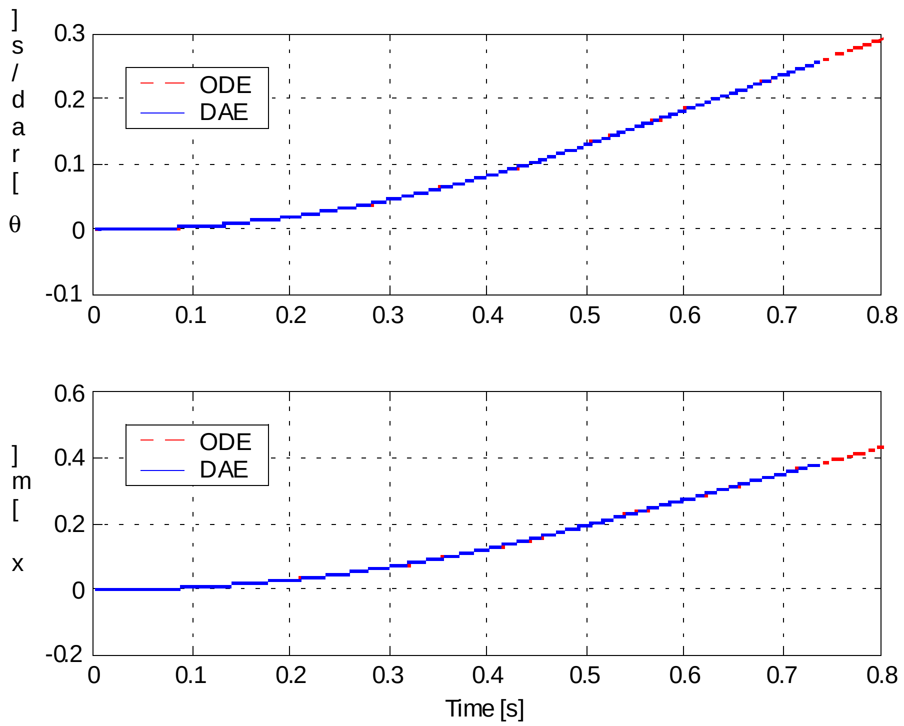

Figure 9. DAE and ODE –simulation result comparison. Displacement variables.

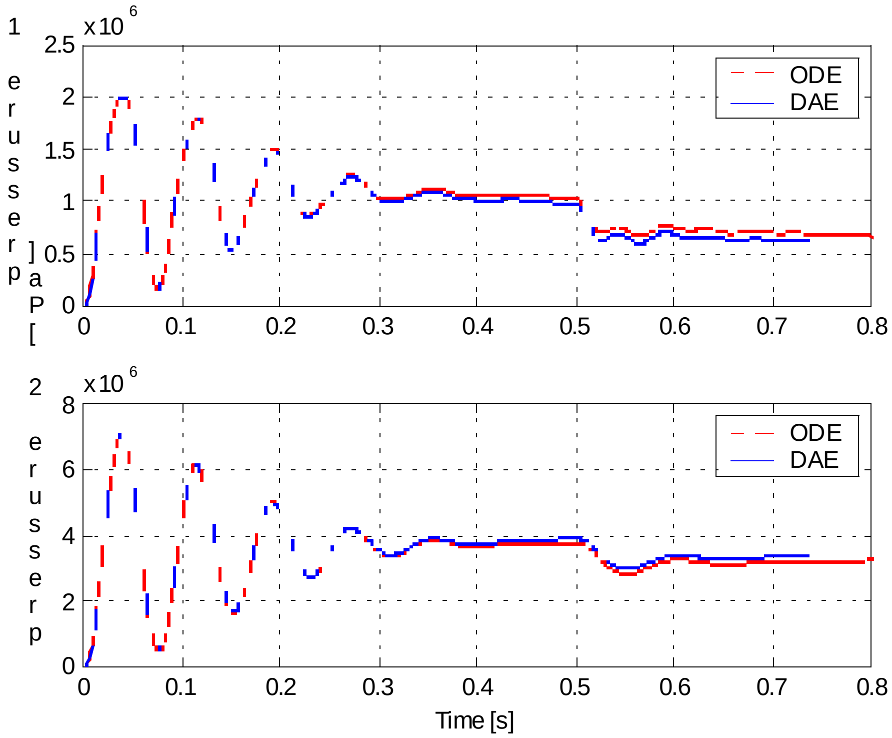

Figure 10. DAE and ODE simulation result comparison. Hydraulic pressures.

APPENDIX A

MATLAB® m-files of ODE model

File *boom .m*, the main program

```
clear all
close all

global A1 A2 myy m1 m2 g I1A I2 L1 L2 r d xp yp B ps dpnom qnom V0 L40

L1 = 3;                 % length of boom 1
L2 = 2;                 % lenght of boom 2
r = 2;                  % position of fastening point of cylinder 2 on boom 1
d = 0.3;                % distance between slider pieces
m1 = 300;               % mass of boom 1
m2 = 200;               % mass of boom 2
I1A = 1/3*m1*L1^2;      % moment of inertia of boom 1 about origin A
I2 = 1/12*m2*L2^2;      % moment of inetria of boom 2 about center of g
xp = 1;                 % (xp,yp) end point of cylinder 2 on ground
yp = -1;
myy = 0.15;             % friction coefficient
A1 = 0.002;             % piston area of cylinder 1
A2 = 0.002;             % piston area of cylinder 2
B = 1.5e9;              % bulk modulus of oil
ps = 25e6;              % supply line pressure, 250 bar
dpnom = 15e6;           % nominal pressure difference, 150 bar
qnom = 0.0025;          % nominal volumetric flow, 1500 liter/min
V0 = 0.001;             % initial volume
g = 9.81;
L40 = sqrt((r-xp)^2+yp^2);

tmax = 2;               % simulation end time

% initial values
x0 = 0; dx0 = 0; Theta0 = 0; dTheta0 = 0; p10 = 0; p20 = 0;
z0 = [ dx0; dTheta0; x0; Theta0; p10; p20 ];

[t,z] = ode45('boomode',[0 tmax],z0);

t = t'; dx = z(:,1)'; dTheta = z(:,2)';
x = z(:,3)'; Theta  = z(:,4)';
p1 = z(:,5)'; p2 = z(:,6)';

% some variables
figure(1)
clf
subplot(2,2,1),plot(t,x),ylabel('x [m]'),grid on
subplot(2,2,2),plot(t,Theta),ylabel('\theta [rad/s]'),grid on
subplot(2,2,3),plot(t,p1/1e5),xlabel('Time [s]'),ylabel('pressure 1 [bar]'),grid on
subplot(2,2,4),plot(t,p2/1e5),xlabel('Time [s]'),ylabel('pressure 2 [bar]'),grid on

% picture of boom
figure(2)
clf
plot(L1*cos(Theta),L1*sin(Theta),'k')
hold on
plot((L1+x).*cos(Theta),(L1+x).*sin(Theta),'k')
line([0 L1*cos(Theta(length(t)))],[0 L1*sin(Theta(length(t)))])
line([ L1*cos(Theta(length(t))) (L1+x(length(t)))*cos(Theta(length(t))) ], ...
   [L1*sin(Theta(length(t))) (L1+x(length(t)))*sin(Theta(length(t)))])
line([xp r*cos(Theta(length(t)))],[yp r*sin(Theta(length(t)))])
plot(0,0,'ro')
plot(L1*cos(Theta(length(t))),L1*sin(Theta(length(t))),'ro')
plot((L1+x(length(t)))*cos(Theta(length(t))), ...
   (L1+x(length(t)))*sin(Theta(length(t))),'ro')
plot(xp,yp,'ro')
plot(r*cos(Theta(length(t))),r*sin(Theta(length(t))),'ro')
grid on
axis equal
```

File *boomode .m*, the odefile

```
function dz = boomode(t,z)

global A1 A2 myy m1 m2 g I1A I2 L1 L2 r d xp yp B ps dpnom qnom V0 L40

y1 = z(1);
y2 = z(2);
x = z(3);
Theta = z(4);
p1 = z(5);
p2 = z(6);

L4 = sqrt((r*cos(Theta)-xp)^2+(r*sin(Theta)-yp)^2);

I = I1A + I2 + m2*(L1-L2/2+x)^2;

MF2 = r*(xp*sin(Theta)-yp*cos(Theta))*p2*A2/L4;
dy2 = (MF2 - m1*g*cos(Theta)*L1/2 - m2*g*cos(Theta)*(L1-L2/2+x))/I;

N1 = (m2*g*cos(Theta) + m2*(L1-L2/2+x)*dy2)*(L2/2-x)/d - I2*dy2/d;
N2 = m2*g*cos(Theta) + m2*(L1-L2/2+x)*dy2 - N1;
N = abs(N1) + abs(N2);
dy1 = (p1*A1 - myy*N - m2*g*sin(Theta))/m2;

if t < 0.5
   f = t;
else
   f = 0.5;
end

x4 = L4 - L40;
dx4 = r*(xp*sin(Theta)-yp*cos(Theta))*y2/L4;

dp1 = B*(sqrt((ps-p1)/dpnom)*qnom*f-A1*y1)/(A1*x+V0);
dp2 = B*(sqrt((ps-p2)/dpnom)*qnom*f-A2*dx4)/(A2*x4+V0);

dz = [ dy1; dy2; y1; y2; dp1; dp2 ];
```

APPENDIX B

DYNAST input-file of DAE model

```
: File BOOM2    Date 13/12/1999    Time 18:12:28
*: boom
*SYSTEM;

L1 = 3;
L2 = 2;
r = 2;
d = 0.3;
m1 = 300;
m2 = 200;
I1 = 1/12*m1*L1**2;
I2 = 1/12*m2*L2**2;
xp = 1;
yp = -1;
myy = 0.15;
A1 = 0.002;
A2 = 0.002;
B = 1.5e9;
ps = 25e6;
dpnom = 15e6;
qnom = 0.0025;
V0 = 0.001;
g = 9.81;
L04 = sqrt((r-xp)**2+yp**2);
f /PULSE/ L2=0.5, TR=0.5, TT=5;

SYSVAR f1,f2,f3,f4,f5,f6,f7,f8,f9,f10,q1,q2,q3,q4,q5,q6,q7,q8,q9,q10,
kappa1,kappa2,kappa3,kappa4,kappa5,kappa6,kappa7,kappa8,
L4,F2x,F2y,x4,dx4,p2,F1x,F1y,x,dx,p1,N1,N2,Fmx,Fmy;

0 = VD.q1 - f1;
0 = VD.q2 - f2;
0 = VD.q3 - f3;
0 = VD.q4 - f4;
0 = VD.q5 - f5;
0 = VD.q6 - f6;
0 = VD.q7 - f7;
0 = VD.q8 - f8;
0 = VD.q9 - f9;
0 = VD.q10 - f10;

0 = m1*VD.f1 + kappa1;
0 = m1*VD.f2 + m1*g + kappa2;
0 = I1*VD.f3 + kappa1*L1/2*sin(q3) - kappa2*L1/2*cos(q3)
+ kappa4 + kappa5*r*sin(q3) - kappa6*r*cos(q3);
0 = m2*VD.f4 - kappa3*tan(q6) - kappa7;
0 = m2*VD.f5 + m2*g + kappa3 - kappa8;
0 = I2*VD.f6 - kappa3*q4*(1+tan(q6)**2) - kappa4
- kappa7*L2/2*sin(q6) + kappa8*L2/2*cos(q6);
0 = - F2x + kappa5;
0 = - F2y + kappa6;
0 = - F1x + Fmx + kappa7;
0 = - F1y + Fmy + kappa8;

0 = q1 - L1/2*cos(q3);          : kappa1
0 = q2 - L1/2*sin(q3);          : kappa2
0 = q5 - q4*tan(q6);            : kappa3
0 = q3 - q6;                    : kappa4
0 = q7 - r*cos(q3);             : kappa5
0 = q8 - r*sin(q3);             : kappa6
0 = q9 -  (q4 - L2/2*cos(q6)); : kappa7
0 = q10 - (q5 - L2/2*sin(q6)); : kappa8

0 = F1x - cos(q6)*p1*A1;
0 = F1y - sin(q6)*p1*A1;
0 = F2x - (q7-xp)/L4*p2*A2;
0 = F2y - (q8-yp)/L4*p2*A2;
0 = Fmx - myy*cos(q6)*(abs(N1)+abs(N2));
0 = Fmy - myy*sin(q6)*(abs(N1)+abs(N2));
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0 = x - (sqrt(q9**2+q10**2)-(L1-L2));
0 = dx - (q9*f9+q10*f10)/sqrt(q9**2+q10**2);

0 = L4 - sqrt((q7-xp)**2+(q8-yp)**2);
0 = x4 - (L4-L40);
0 = dx4 - ((q7-xp)*f7+(q8-yp)*f8)/L4;

0 = VD.p1 - B*(sqrt((ps-p1)/dpnom)*qnom*f(TIME)-A1*dx)/(A1*x+V0);
0 = VD.p2 - B*(sqrt((ps-p2)/dpnom)*qnom*f(TIME)-A2*dx4)/(A2*x4+V0);

0 = N1 – ((m2*g*cos(q6)+m2*(L1-L2/2+x)*VD.f6)*(L2/2-x)/d - I2*VD.f6/d);
0 = N2 – (m2*g*cos(q6) + m2*(L1-L2/2+x)*VD.f6 - N1);

*TR;
tr 0 2.5;
INIT q1=1.5, q2=0, q3=0, q4=2, q5=0, q6=0, q7=2, q8=0, q9=1, q10=0, p1=0, p2=0;
PRINT(1001) q3,x,p1,p2;
RUN;

*END;
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